



Computer–Aided Design of the Critical Speed of Shafts

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ABSTRACT: A computer aided design software for the analysis of the critical speed of shaft, is developed and presented in this work. The software was designed using the principles of object oriented programming, and implemented with the Microsoft Visual Basic Language. The package was tested on a number of benchmark design problems. The solutions obtained were highly accurate. Also, the software exhibited very high efficiency. To improve on the method of presenting these solutions, graphical features were incorporated. This enhances the ability to visualize results. @ JASEM

All rotating shafts, even in the absence of external load, deflect during rotation. The magnitude of the deflection depends upon stiffness of the shaft and its supports, the total mass of shaft and attached part, the unbalance of the mass with respect to the axis of rotation, and the amount of damping in the system. As long as the deflections are minimal, the shaft can still operate satisfactorily. However, as the speeds increase, the shafts become unstable. Thus, there is the need to determine these critical speeds at which instability sets in. With the deflection, now considered as a function of speed, only the lowest

(first) and occasionally and the second are of interest to the designer, the others will usually be so high as to be well out of range of the operating speed. At the first critical speed, the shaft will bend to the simplest shape possible. At the second critical speed it will bend to the second simplest shape possible etc. for example a shaft supported at its end and having a large (compared to shaft) masses attached, will bend according to the configurations shown in Figures 1 and Figure 2 at the first and second critical speed respectively.

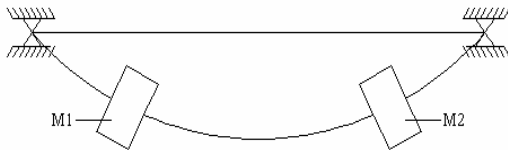


Fig.1: Loading configuration at the first critical speed

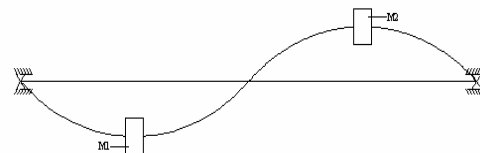


Fig. 2: A variant of the Loading configuration at the first critical speed

We wish to state that to the best of our knowledge there is addressed the problem of reducing the computational no documented computer-aided design work for resolving difficulties usually encountered in manually designing for the critical speed of shafts. The authors would have the critical speeds of shafts. And then provided a visual conducted a comparative assessment of our results with display of the solutions, so as to easily and speedily any that was available. But the results presented here reinterpret these solutions. The natural frequency of the highly accurate as they were solutions to bench mark shaft is very nearly the same as the critical speed and is examples and problems adopted from (Hall et al, 2000). usually taken as the same thing. There is a difference This work is a continuation of efforts by the authors to usually quite small due the gyroscopic action of the mass. develop software for the design of machine components.

Earlier works had considered the design of: some types of **DESIGN OF SOFTWARE MODEL** gears (helical, and spur), design of Rolling Bearings, In designing the software, all the parameters machine vibrations design see (Akpobi, 1998; Akpobi; necessary for proper analysis of the critical speed of Ardey, 2002; Akpobi; Airiohuodion 2005). And quite shafts, were carefully assembled, and used in recently, software for the design of Flywheels, was developing the programme that forms the software in developed and reported see (Akpobi; Lawani, 2006). This is present work. In the software, the allowable design for the critical speeds of shafts, is well expounded loads were defined and the corresponding induced in standard machine design texts, see (Hall et al, 2000; obtained. The concept of object oriented Shigley; Mischke, 2001). In this work the problem we programming technique was employed in the design focused on, was to develop and implement a computer of the software. This principle was then implemented software that would greatly enhance an accurate and using Microsoft Visual Basic language (Microsoft efficient design for the critical speeds of shafts. Also we

Incorporated, 1998; Byron, 2002). These design parameters are presented as follows:

SHAFT WITH SINGLE ATTACHED MASS

If the shaft mass is small compares to the attached mass, the first critical speed (W_c) can be calculated approximately as:

$$W_c = \sqrt{k / m} \quad 1$$

Where k shaft spring constant, and m is mass

Also

$$W_c = \sqrt{g / \delta} \quad 2$$

where g is acceleration due to gravity (m/s^2) and δ is static deflection (m)

SHAFT OF CONSTANT CROSS SECTION

If the shaft is simply supported at the end, with no mass involved other than that of the Shaft itself, the first critical speed will be

$$W_c = \sqrt{5 / 4(g / \delta_{max})} \quad 3$$

where δ_{max} is the maximum static deflection (m)

SHAFT OF NEGLIGIBLE MASS CARRYING SEVERAL CONCENTRATED LOAD

For shaft of negligible mass carrying several concentrated masses, the first critical speed is obtained using the Rayleigh Ritz equation (Equation 4).

$$w_c = \sqrt{\frac{g_1^j \sum (w_n \delta_n)}{j_1 \sum (w_n \delta_n)}} \quad 4$$

where δ_n is static deflection (m) at the nth mass

w_n is the weight of the nth mass and j is the total number of masses.

The Rayleigh Rita equation above can be used for estimating the critical speed of shafts with distributed loads. The method involves breaking the distributed masses into a series of masses $m_1, m_2, m_3, \dots, m_n$, after which each of the masses is taken to be a concentrated load having its weight acting at its center.

THE DUNKERLEY EQUATION

Another approximation for the first critical speed of a multiple mass system is

$$1 / w_c^2 = 1 / w_1^2 + 1 / w_2^2 + 1 / w_3^2 + \dots \quad 5$$

where w_1 is the first critical speed with only mass 1 present, w_2 is the first critical speed with only mass 2 present, e.t.c.

Both the Rayleigh- Ritz and the Dunkerley equations are approximations to the first natural frequency of

vibration, which is assumed to be nearly equal to the critical speed of vibrations. In general, however, the Dunkerley equation underestimates while the Rayleigh- Ritz equation overestimates the natural frequency.

HIGHER CRITICAL SPEEDS FOR MULTIPLE MASS SYSTEM

This requires much more extensive computations than is necessary for the determination of the lowest (first) critical speed. Several methods have been developed to for this purpose. For a two mass system is given as, we have:

$$1/w^4 - (a_{11}m_1 + a_{22}m_2)1/w^2 + (a_{11}a_{22} - a_{12}a_{21})w_1m_2 = 0 \quad 6$$

where a_{ij} are the influence coefficients,

This is a quadratic equation having two positive roots $1/w_1$ and $1/w_2$

For any multi-mass system, the frequency equation is obtained by setting the following determinants equal to zero.

$$\begin{vmatrix} (a_{11} - \frac{1}{w^2}) & (a_{12}m_2) & (a_{13}m_3) & \dots \\ (a_{12}m_1) & (a_{22}m_2 - \frac{1}{w^2}) & (a_{23}m_3) & \dots \\ (a_{31}m_1) & (a_{32}m_2) & (a_{33}m_3 - \frac{1}{w^2}) & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix} \quad 7$$

PROGRAMME DESCRIPTION

The program was designed and implemented using Microsoft Visual Basic object oriented programming language. The structure of the program is such that there are three stages. These stages include the input stage, analysis and output stage.

INPUT STAGE

At this stage, the user enters the required information about the Critical speed of the shaft, in the input interface form. The programme is well written in such a way that the software requires minimal input to carryout its analysis.

ANALYSIS

After the required data is in entered, the analysis carried out. Once the data is entered, the software computes all the parameters required in analyzing the critical speed of shaft in less than 2 seconds depending on the speed of the processor.

OUTPUT

The software is designed such that the critical speed parameters are outputted numerically and graphically with the accompanying description of the result.

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PROGRAMME'S ALGORITHM OR PSEUDO-CODE

The software was designed using the following algorithm or the programme's pseudo-code:

If Option.value = 1 Then
 Display introduction
 End If

If Option.value = 2 Then
 If the system has only one mass, Then
 Input known parameters from: K, m. compute desired parameters using Equations 1 and 2
 Compute values for initial speed and generate plots of initial speed against mass.
 End If
 End If

If Option.value = 3 Then
 If the uniform shaft carries no mass Then
 Input known parameters δ_{max}
 Compute desired parameters using equation 3
 Compute values for the critical speed of the shaft and generate a plot of critical speed against δ_{max} .
 End If
 End If

If Option.value = 4 Then
 If deflection of the mass are known then
 Input known parameters:
 $\delta_1, \delta_2, \delta_3, \dots, \delta_n, w_1, w_2, w_3, \dots, w_n$
 Compute desired parameters using equation 4
 Compute value for the critical speed of the shaft and generate a plot of critical speed against w_1 or w_2 .
 Else If the influence coefficient is known Then
 Input known parameters: $a_{11}, a_{22}, a_{21} = a_{12}, w_1, w_2, \dots, w_n$
 Compute desired parameters using equation 6
 End If
 End If
 End If

If Option.value = 5 Then

If the system has only one mass Then
 If the system has spring support Then
 If considering a rigid support Then
 Input known parameters: K, δ_b, w, l_1, l_2
 Compute desired parameter using Equations 4 and 5
 Else If considering a spring support then
 Click the check.value = 1
 Compute desired parameters and obtain value for critical speed and the effect of spring support on the system.
 End If
 End If
 End If
 End If
 End If

EXAMPLES TO ILLUSTRATE THE USE OF THE SOFTWARE

To illustrate the effectiveness and numerical accuracy of the software, the following Examples were considered.

EXAMPLE 1

Calculate the critical speed of rotating shaft carrying a mass M_1 of 50kg having a shaft spring constant of 250Nm, as shown in Fig. 4.

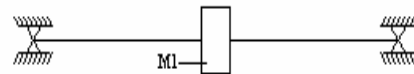


Fig. 4: Loading configuration for Example 1

SOLUTION

The out put from the software for this analysis on inputting values for Mass of 50kg, spring constant of 250Nm is as follows:

The critical speed is 2.236 rad/s

The software also gave as output a graph of:
 Varying critical speed (rad/s) against mass (kg)

The screen shot of the solution in shown Fig. 5

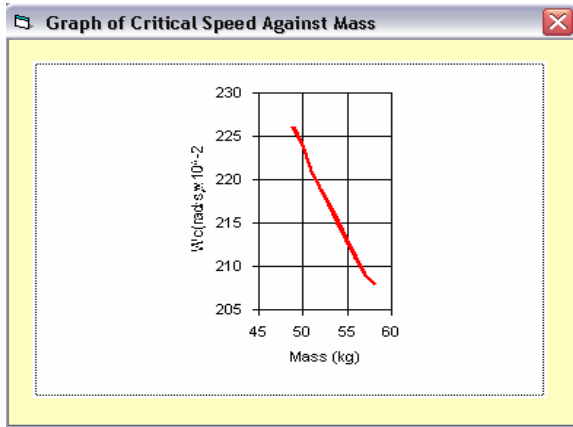


Fig. 5: Screenshot Of The Result For Example 1

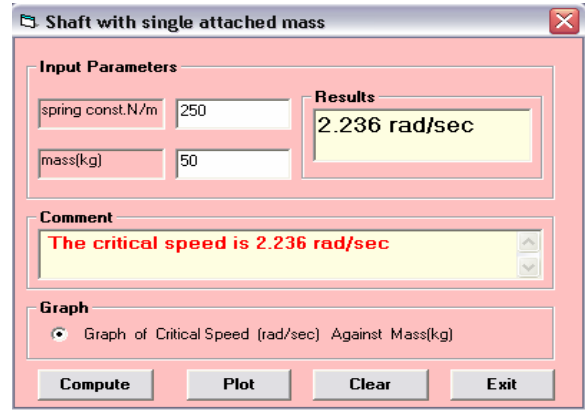


Fig. 6: Screenshot of the graph of critical speed against mass for Example 1

EXAMPLE 3

Compute the critical speed of a rotating shaft with no mass attached to it having a maximum static deflection of 0.03mm caused by a uniformly distributed load

SOLUTION

The out put from the software for this analysis on inputting value for Maximum static deflection of 0.03mm is as follows: The critical speed is 6.065(rad/s)

The software also gave as output a graph of:
Varying critical speed (rad/s) against Maximum deflection (mm)

The screen shot of the solutions are shown in Figs. 7 and 8

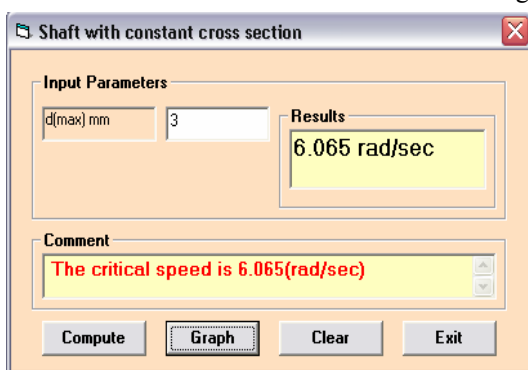


Fig. 7: screenshot of the result for Example 2.

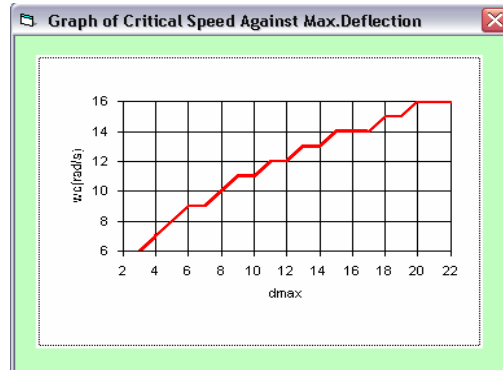


Fig. 8: Screenshot of the graph of critical speed against maximum deflection for Example 2

EXAMPLE 3

The shaft shown in Fig. 8 has attached to it a gear weighing 220N and flywheel weighing 440N, while

the static deflection δ_1 and δ_2 have been found to be 0.03mm and 0.008 mm respectively. Determine the critical speed ignoring the mass of the shaft.

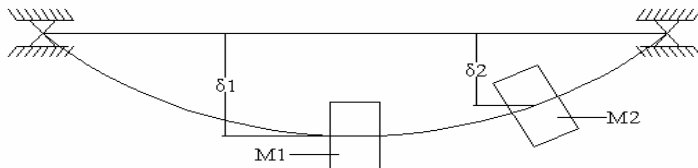


Fig. 9: Loading configuration for Example 3

SOLUTION

The out put from the software for this analysis on inputting values for Gear of weight, 220N, Flywheel of weight, 440N, static deflection for gear is 0.03 and for flywheel is 0.008 is as follows:

The software also gave as output a graph of:

Varying critical speed (rad/s) against weight 1 (N)

Varying critical speed (rad/s) against weight 2 (N)

The screen shot of the solutions are shown in Figs. 10-12

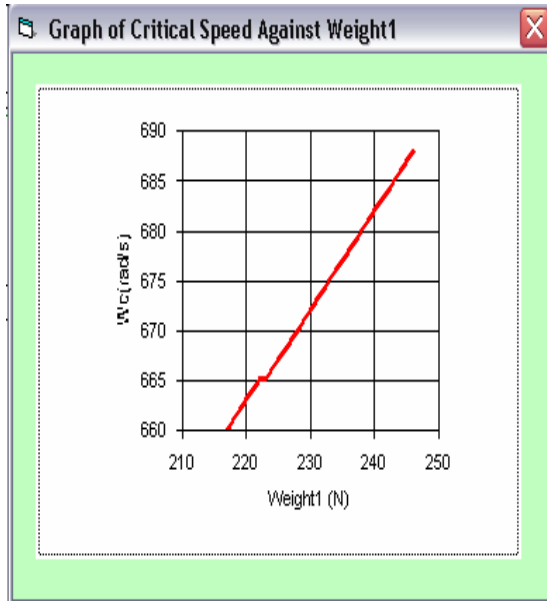


Fig. 10: Screenshot of the graph of critical speed against weight 1 for Example 3

Input Parameters			
Weight 1(N)	220	Deflection1(mm)	0.03
Weight 2(N)	440	Deflection2(mm)	0.008
Weight 3(N)		Deflection3(mm)	
Weight 4(N)		Deflection4(mm)	
Weight 5(N)		Deflection5(mm)	

Number conc. masses

2 masses

3 masses

4 masses

5 masses

Results

662.547259895723

Comment

The critical speed is 662.547259895723(rad/sec)

Graph

Graph of critical speed(rad/sec) against weight1(N)

Graph of critical speed(rad/sec) against weight2(N)

Fig.11: screenshot of the result for Example 3

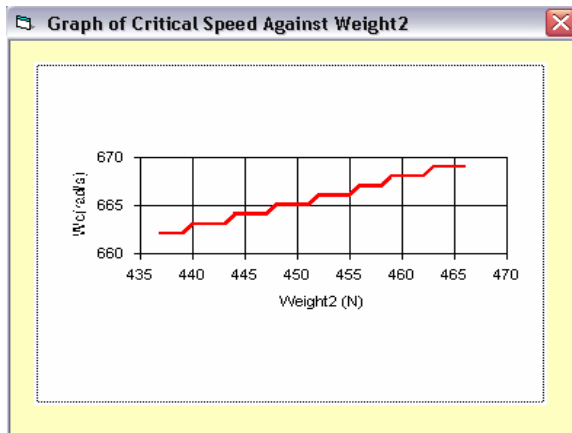


Fig. 12: Screenshot of the graph of critical speed against weight 2 for Example 3

EXAMPLE 4

The masses m_1 and m_2 , attached to the shaft of Fig.12 weighing 625N and 270N respectively through a deflection analysis, the influence coefficients of the shaft has been found to be

$$A_{11} = 1.142E-8 \text{ m/N}$$

$$A_{22} = 6.853E-8 \text{ m/N}$$

$$A_{12} = 2.284E-8 \text{ m/N}$$



Fig. 13: Loading configuration for Example 4

SOLUTION

The out put from the software for this analysis on inputting values for Weight 1 of 625N, weight 2 of 270N, Influence coefficients of $A_{11}= 1.142E-8$ m/N $A_{22}= 6.853E-8$ m/N and $A_{12}=21=2.284E-8$ m/N is as follows:
The critical speed is 618.544rad/s

The screen shot of the solution is shown in Fig. 14

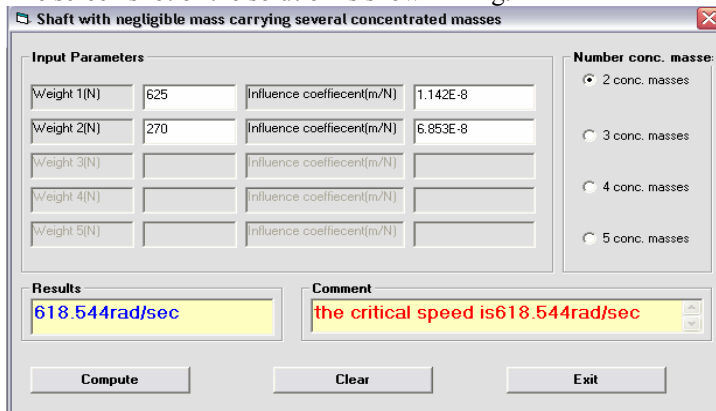


Fig.14: screenshot of the result for Example 4.

EXAMPLE 5

The bearing support for the shaft is shown in Fig.12. It has flexibility equivalent to a spring constant of k 44MN/m in any direction perpendicular to the shaft

axis. Due to bending, the shaft itself has a deflection δ of 0.046mm under the 1350N load. What effect does the flexibility of the support have on the critical speed?

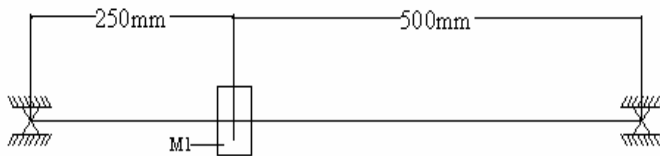


Fig. 15

SOLUTION

The out put from the software for this analysis on inputting values for Spring constant of 44MN/m, Deflection of 0.046mm, load of 1350N, Distance between support A and the mass is $(L_1) = 250$ mm, while that between support B and mass $(L_2) = 500$ mm is as follows:

The reaction at the support A is 900 N and the reaction at support B is 450 N.
The critical speed due to rigid support is 461.802 rad/s.

The screen shot of the solution is shown in Figs. 16 and 17

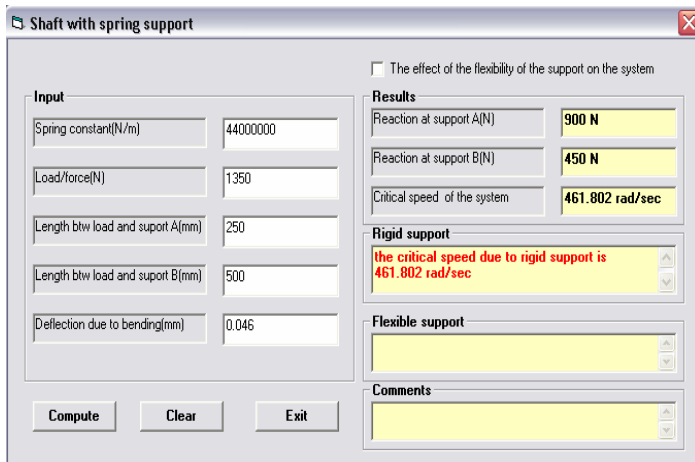


Fig. 16: screenshot of the result for Example 5

The software also produced as output:

The critical speed due to flexible support is 394.464 rad/s

The flexibility of the support reduces the critical speed by 15.0%

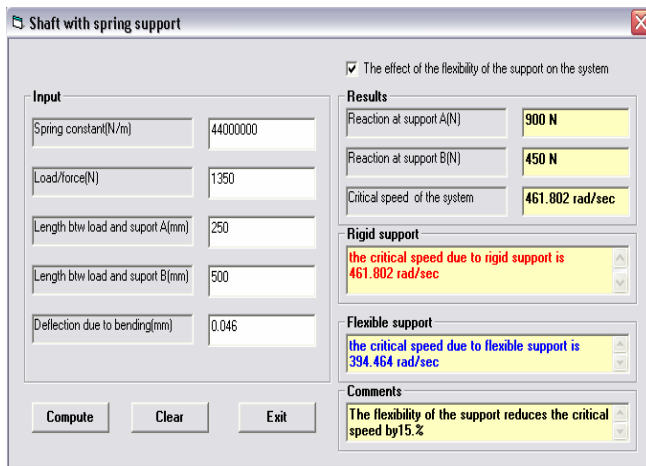


Fig. 17: screenshot of the result for Example 5.

Discussion: This software was tested for this paper using 5 Examples, in which the problems on critical for systems with single mass, multi-mass having rigid support or spring support and systems with known static deflections, known influence coefficient were analyzed. The results displayed by the software depend on the problem definition and provides the designer with the exact numerical values required to design for the critical speeds of shaft.

The software also outputted graphical features for the following:

- Critical speed (rad/s) against mass (kg)
- Critical speed (rad/s) against maximum deflection (mm)
- Critical speed (rad/s) against weight 1 (N)
- Critical speed (rad/s) against weight 2 (N)

The results for the various Examples shown in Figs. 5-17 are highly accurate and even better than results obtain from the text (Hall, et al) from which they were adopted. The accuracy obtained was due to the

use of double precision (12 decimal places) variables in storing computed values before formatting final answers to the users designed level of decimal places.

Conclusion: We have presented in this work, a computer-aided design software that allows for easy design and analysis of the critical speed of shaft. The number of practical problems formulated and solved accurately using this software, shows that it is flexible, accurate, and robust. The graphical features incorporated in the software, also enhances the ability of the user to visualize and interpret solutions easily.

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